9/19

Ex 50

$$\sum_{k=0}^{99} {}_{99}C_k = 2^{99}$$

${}_{99}C_k = {}_{99}C_{99-k}$ より,

$$2\sum_{k=0}^{49} {}_{99}C_k = 2^{99}$$

$$\therefore\ 2^{98}$$

8

Ex 51

(1)
$$a_n = \sum_{k=0}^{n} k\, {}_nC_k$$

$$= \sum_{k=0}^{n} n\, {}_{n-1}C_{k-1}$$

$$= n\sum_{k=1}^{n} {}_{n-1}C_{k-1}$$

$$= n \cdot 2^{n-1}$$

(2)
$$\sum_{k=0}^{n} k^2\, {}_nC_k = \sum_{k=0}^{n} k \cdot n \cdot {}_{n-1}C_{k-1}$$

$$= n\left\{\sum_{k=0}^{n} (k-1)\, {}_{n-1}C_{k-1} + \sum_{k=0}^{n} {}_{n-1}C_{k-1}\right\}$$

$$= n\left\{(n-1)\sum_{k=0}^{n} {}_{n-2}C_{k-2} + 2^{n-1}\right\}$$

$$= n(n-1)\,2^{n-2} + n\,2^{n-1}$$

Pra 36

$$\sum_{k=0}^{n} k \cdot {}_nC_k\, p^k q^{n-k}$$

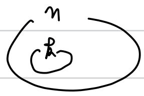
$$= \sum_{k=0}^{n} n\ {}_{n-1}C_{k-1}\, p^k q^{n-k}$$

$$= np \sum_{k=0}^{n} {}_{n-1}C_{k-1}\, p^{k-1} q^{n-k}$$

$$= np \sum_{k=0}^{n-1} {}_{n-1}C_k\, p^k q^{n-k}$$

$$= np \cdot (p+q)^{n-1}$$

$$= np$$



Ex 52

$${}_2C_2 + {}_3C_2 + \cdots + {}_nC_2 \qquad {}_nC_k = {}_{n-1}C_{k-1} + {}_{n-1}C_k$$

$$= \sum_{k=2}^{n} {}_kC_2$$

コンビネーション単体の和だから，定義で考える．

$$= \sum_{k=2}^{n} \frac{k(k-1)}{2}$$

$$= \frac{1}{2} \sum_{k=2}^{n} k(k-1) \qquad a\{(k+1)-(k-2)\} = 1$$

$$= \frac{1}{2} \sum_{k=2}^{n} \frac{1}{3}\{(k+1)k(k-1) - k(k-1)(k-2)\}$$

$$= \frac{1}{6}\{(n+1)\cdot n\cdot(n-1) - 0\}$$

$$= \frac{1}{6} n(n^2-1)$$

Ex 38

$${}_nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_r$$

より，

$${}_{n+1}C_{r+1} - {}_nC_{r+1} = {}_nC_r$$

$${}_{n+1}C_{r+1} - {}_rC_{r+1} = \sum_{k=r}^{n} {}_nC_r$$

$$\therefore\ {}_{n+1}C_{r+1} = \sum_{k=r}^{n} {}_nC_r$$